Microcosm to Macrocosm Via the Notion of a Sheaf (Observers in Terms of t-Topos)

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The fundamental approach toward matter, space and time is that particles (either objects of macrocosm or microcosm), space and time are all presheafified. Namely, the concept of a presheaf is most fundamental for matter, space and time. An observation of a particle is represented by a morphism from the observed particle (its associated presheaf) to the observer (its associated presheaf) over a specified object (called a generalized time period) of a t-site (i.e. a category with a Grothendieck topology). This formulation provides a scale independent and background space-time free theory (since, for the t-topos theoretic formulation, space and time are discretely defined by the associated particle, whose particle-dependency is a consequence of quantum entanglement.). It is our basic scheme that the method of t-topos may provide a device for understanding and concrete formulation of macro-object and micro-object interconnection as morphisms in the sense of t-topos.

1. Introduction

The concept of a sheaf appeared in 1940's by Leray in algebraic topology and by K. Oka in the theory of analytic functions in several complex variables. In 1960's, theory of sheaves (cohomology of sheaves) made tremendous development in algebraic geometry, especially by Grothendieck and in algebraic analysis by M. Sato and M. Kashiwara. The notion of a sheaf has been used to obtain global information from local data. The definition of a sheaf is a contravariant functor from a category with a Grothendieck topology, called a site, to a category satisfying the sheaf axiom as indicated in [1], [2], [3]. The notion of a topos, i.e. a category of (pre-) sheaves over a category with a Grothendieck topology has been used to study quantum gravity by Isham as in [4], Mallios Rapitis as in [5], Guts Grinkevisch as in

[6]. Kato as in [7] and [8] and others. One of the challenges for such a theory is to explain the physical reality by building a micro-macro theory, which is scale independent, (e.g. assigning real numbers or complex numbers for measuring quantities), and background spacetime free. In quantum gravity, such a theory needs to be a background spacetimeless theory. In our theory of t-topos, we have obtained certain levels of success as shown in [7], [8], [9], [10], [11]. For a general survey on the foundations of physics based on hidden variable theories, see [12], where the reports on various theories and ideas with direct experimental applications are thoroughly given.

2. The Very Small to not so Small and the Large

We often divide the physical world into the microcosm where quantum physics is applied and the macrocosm where classical (or/and relativistic) mechanics is applied. One of the fundamental approaches to obtain a unified theory is to build the global (macro) object information from studying local (micro) objects. The inadequacy of the traditional continuum-based models (e.g. classical differential geometry based on either real numbers or complex numbers) as final physical theories has been pointed out by various authors. Our approach is to apply the concept of a (pre)sheaf as a candidate for such a unifying theory. Hence, the fundamental reality is described by a discrete concept. Via the notions of categories and sheaves, we need to describe the concepts of wave-particle duality, measurement and quantum entanglement in microcosm and the concepts of a light-cone and the gravitational effect on spacetime in macrocosm. (See the above references.) Here is our initial step toward such a unification theory of the very small, not so small and the large. Some of the results have been obtained toward this goal in [7], [8], [9], [10].

By definition, a category consists of objects and morphisms satisfying certain axioms as in the above references. Let us recall that a presheaf is a contravariant functor from a site S (a category with a Grothendieck topology. See [1], [2], or [3].) to a category. The basic notion is the triple of presheaves (m, κ, τ) associated with a particle, space and time, respectively. Note that presheaves (κ ,τ) are depending upon the particle m. Namely, the space and time depend upon locally m. Recall that this dependency is a consequence of quantum entanglement. (See [7] and [8] for the assertion.) Let m be a presheaf over a site S, i.e. m $\in$Ob($\hat{S}$), i.e. m is an object of the category $\hat{S}$of presheaves over S. (For definitions of presheaves, sites and toposes and the notations, see [1], [2], or [3].) Note also that in [7] and [8], the term a "t-site" is used when it is referred to in the theory of t-topos.

An object of t-site S is said to be a generalized time (period). There are two types of ur-wave states in the theory of t-topos : first, when the particle presheaf m is not observed, and secondly, when there are more than one choice in objects in the t-site S (as in the case of well-known double-slit experiment), then m is said to be the ur-wave state. Next, we will consider the case when m is in the ur-particle state. (For the double-slit application of t-topos and the duality, see the earlier mentioned papers [9] and [12].) That is, when m is observed by P $\in$Ob($\hat{S}$), by definition, there exists a generalized time period V in t-site S so that there exists a morphism s(V) from m (V) to P(V), (see [7]). However, note that a particle can be in the ur-particle state as long as an object in the t-site is specified, regardless whether the particle is observed or not. The notions of microlocalizations of m and V in t-topos theory are decompositions of m and V as described in [8]. That is, write m as a direct product of finitely many sub-presheaves m_i, namely, m $= \prod m_i$, and for V it is to consider a covering $\{V_j\}$ of V, that is, { V$\leftarrow V_j$}, (see [8]). When m is observed by P over V, one obtains no information of m_i. However, when a microcosm-object m_i is observed by P over a generalized time period V_j, there exists a morphism s(ij) from $m_i(V_j)$ to P(V_j). Note that m may not be defined over V_j. Namely, obtaining an information locally during the generalized time period does not imply that the macro-object m can be observed by P over V_j. In general, one cannot compose the morphisms s(ij) with the canonical projection $\pi(V_j)$: m(V_j) $\to m_i(V_j)$ to obtain the morphism from m(V_j) to P(V_j). This is one of the roots of the local-global phenomena: why studying an individual particle m_i does not provide the global information of m $= \prod m_i$.

When m is observed by P over generalized time periods V first and later U in S. Then there exists a morphism ρ from V to U in S. (See [7]) Then there exist morphisms from m(V) to P(V) and m(U) to P(U), respectively For each factorization $V \xrightarrow{\rho'} W \xrightarrow{\rho''} U$ of the morphism $V \xrightarrow{\rho}$ U in the t-site S, there corresponds a ur-particle state m(W) of m. For all the possible factorizations of the morphism of ρ, the presheaf m induces funtorially the corresponding factorizations $m(V) \xleftarrow{m(\rho')} m(W) \xleftarrow{m(\rho'')} m(U)$ of $m(V) \xleftarrow{m(\rho)} m(U)$. Those possible "paths" via {m(W)} between m(V) and m(U) represent the Feynman paths in the classical sense (See Section 3.1 in [9]). Notice also that the directions of arrows are reversed when evaluated at the presheaf m. This is because the presheaf m is a contravariant functor by definition. One can generalize the above by formulating its relativistic version by the notion given in [8].

3. Conclusion

The language of category theory, sheaf theory, i.e. a topos theory can give qualitative analysis in physics, especially in quantum gravity as in [4], [5], [8]. In order to obtain quantitative results, the notion of scaling needs to be introduced. Traditionally, the fields of real numbers and complex numbers have been used for scaling. Some have attempted to use the ring of p-adic integers instead of real or complex numbers. Hence, the categorical approach alone is not sufficient for applications to physics. However, one should have at least one theory formulating both microcosm and macrocosm, i.e., a theory on quantum gravity, with one mathematical model. Our choice is a category of presheaves from a (t-) site to a product category, which is scale independent and background spacetime free.

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